The Half-Life of a Bouncing Ball

INTRODUCTION

This investigation asks the question of whether the height of a bouncing ball displays exponential decay and, if so, what is the half-life of the height?

The **independent variable** is the bounce number. The 'life' of a bouncing ball is measured as the 1^{st}, 2^{nd}, 3^{rd}, etc. bounce number. This is a counting number, a pure number with no units and no uncertainties.

The **dependent** variable is the rebound height, *H*, the height reached between bounces. To measure *H*, the time ΔT between consecutive bounces is measured and *H* calculated from $H = \frac{1}{2}gt^2$ where $t = \frac{1}{2}\Delta T$. The $\frac{1}{2}$ comes from the face that ΔT is the time up to the rebound height plus the time down from the rebound height. It is far more accurate to measure the time interval, and then calculate the height, than it would be to try to measure the rebound height of a moving bouncing ball. There is no significant uncertainty in the calculated height as it is based on a very precise timing mechanism with the computer and interface.

The **controlled** variables include using the same surface and the same ball. The initial drop height is not relevant because the first bounce height is calculated by the time interval between the first impact and the second impact. If the ball moves off the vertical while bouncing, then the data is rejected. Hence a controlled variable is that the bouncing stays more or less along the vertical.

GATHERING DATA

The time ΔT is determined by recording the impact sound of a bouncing ball. Time intervals are read off a graph of sound pressure against time. A number of trials were made and it was found that a ping-pong ball made the cleanest (least noise) sound for the computer to record. A number

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of trials from various initial drop heights were made, and it was found that the best drop height was about 60 cm.

The microphone was connected to the Vernier *Lab Pro* interface and then this was connected to the computer. The Vernier *LoggerPro* 3.4.1 software automatically sensed the microphone and displayed graph axes of sound level against time. The settings were pre-set.

Figure 1 shows a sample of the raw data of sound intensity (in arbitrary units) and time measurements (in seconds).

Figure 1: Raw Data: Sound Pressure (units) and Time (s)

	Time (s)	Sound Pressure (arbitrary)	Offset Pressure (arbitrary)
1	0.000	2.684	0.000
2	0.001	2.684	0.000
3	0.002	2.684	0.000
4	0.003	2.683	-0.001
5	0.004	2.684	0.000
6	0.005	2.684	0.000
7	0.006	2.684	0.000
8	0.007	2.684	0.000
9	0.008	2.684	0.000
10	0.009	2.683	-0.001
11	0.010	2.683	-0.001
12	0.011	2.684	0.000
13	0.012	2.684	0.000
14	0.013	2.685	0.001

This data is then automatically graphed.

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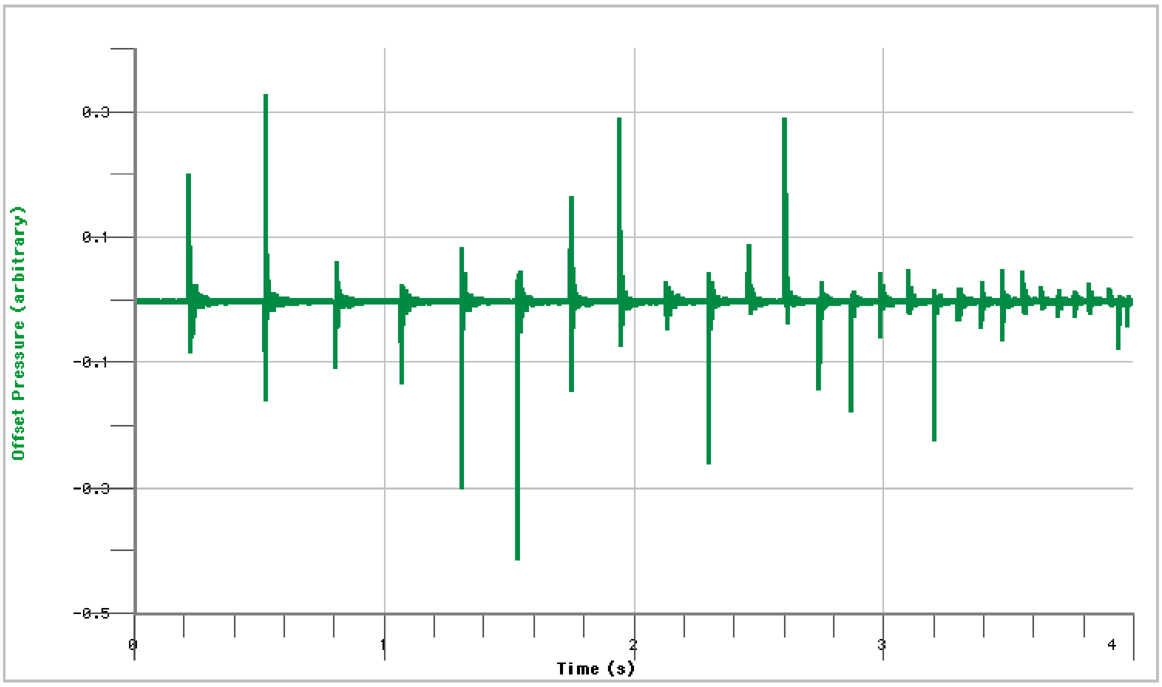
Figure 2: Graph of Sound Pressure (units) against Time (s)



The computer calculates the consecutive times for the first peaks of each bounce and uses this to calculate the rebound height H and the natural logarithms of the height H. A value of $g = 9.81 \mathrm{m\,s^{-2}}$ is used but as this is a constant through out the experiment, it could have been normalized, i.e. $g = 1$.

See the data table below, Figure 3, and details of sample calculations.

Figure 3: Processed Data

	N	Total Time (s)	Delta Time (s)	Height (m)	ln (Height)	H(n+1) / H(n)
			Data Set			
1	0	0.213				
2	1	0.518	0.305	0.114	-2.173	
3	2	0.799	0.281	0.097	-2.337	0.851
4	3	1.061	0.262	0.084	-2.477	0.866
5	4	1.303	0.242	0.072	-2.636	0.857
6	5	1.529	0.226	0.063	-2.773	0.875
7	6	1.741	0.212	0.055	-2.900	0.873
8	7	1.938	0.197	0.047	-3.047	0.855
9	8	2.124	0.186	0.042	-3.162	0.894
10	9	2.292	0.168	0.035	-3.366	0.833
11	10	2.452	0.160	0.031	-3.463	0.886
12	11	2.599	0.147	0.026	-3.633	0.837
13	12	2.735	0.136	0.023	-3.788	0.885
14	13	2.861	0.126	0.019	-3.941	0.826
15	14	2.981	0.120	0.018	-4.039	
16	15	3.092	0.111	0.015	-4.195	
17	16	3.195	0.103	0.013	-4.344	
18	17	3.294	0.099	0.012	-4.423	
19	18	3.384	0.090	0.010	-4.614	
20						

Delta Time Calculation:

$$\text{Delta Time} = \text{Total Time}_{\text{N+1}} - \text{Total Time}_{\text{N}} \quad \rightarrow \quad \Delta T_{\text{N to N+1}} = T_{\text{N+1}} - T_{\text{N}}$$

For example, the interval from N = 3 to N = 4:

$$\Delta T_{3\rightarrow 4} = 1.303\text{s} - 1.061\text{s} = 0.242\text{s}$$

Rebound Height Calculation:

$$\text{Rebound Height} = H = \frac{1}{2}gt^2 = \frac{1}{2}g\left(\frac{\Delta T}{2}\right)^2 = \frac{g\Delta T^2}{8} = \frac{\left(9.81\text{m s}^{-2}\right)\Delta T^2}{8} = 1.22625 \times \Delta T^2$$

For example, H for interval $\text{N} = 3 \rightarrow \text{N} = 4$:

$$H = 1.22625\left(\Delta T_{3\rightarrow 4}\right)^2 = 1.22625\left(0.242\text{s}\right)^2 = 0.071814\text{m} \approx 0.072\text{m}$$

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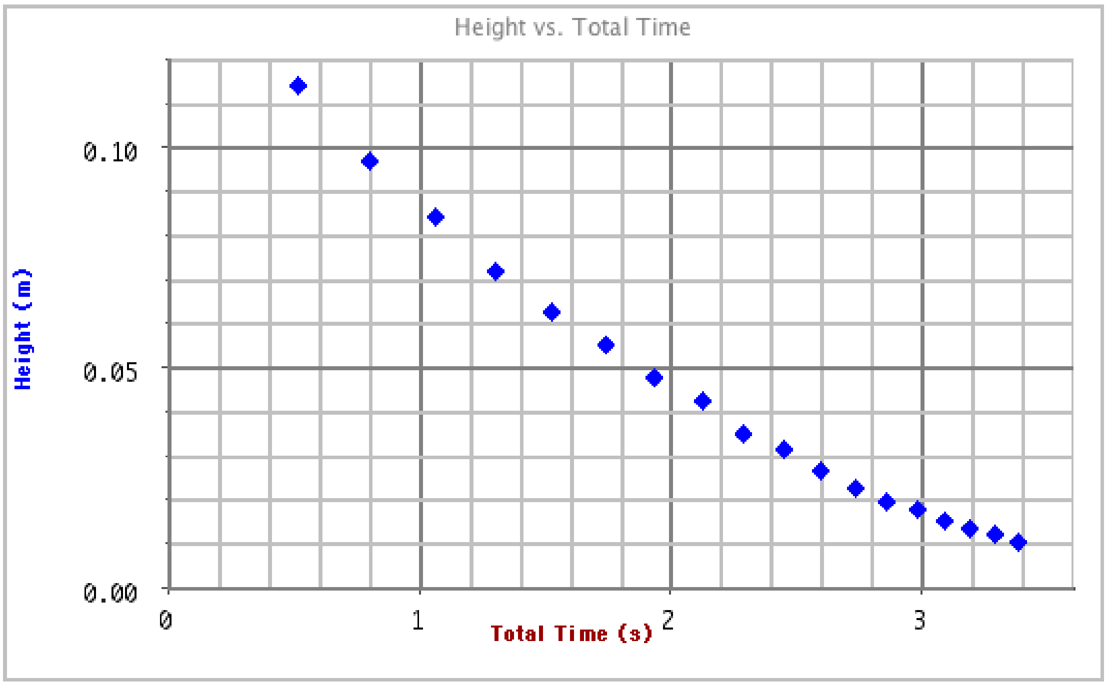
Natural Logarithm of Height Calculation:

$$\ln h_{3\to4} = \ln(0.071814) \approx -2.477$$

ANALYSIS OF DATA

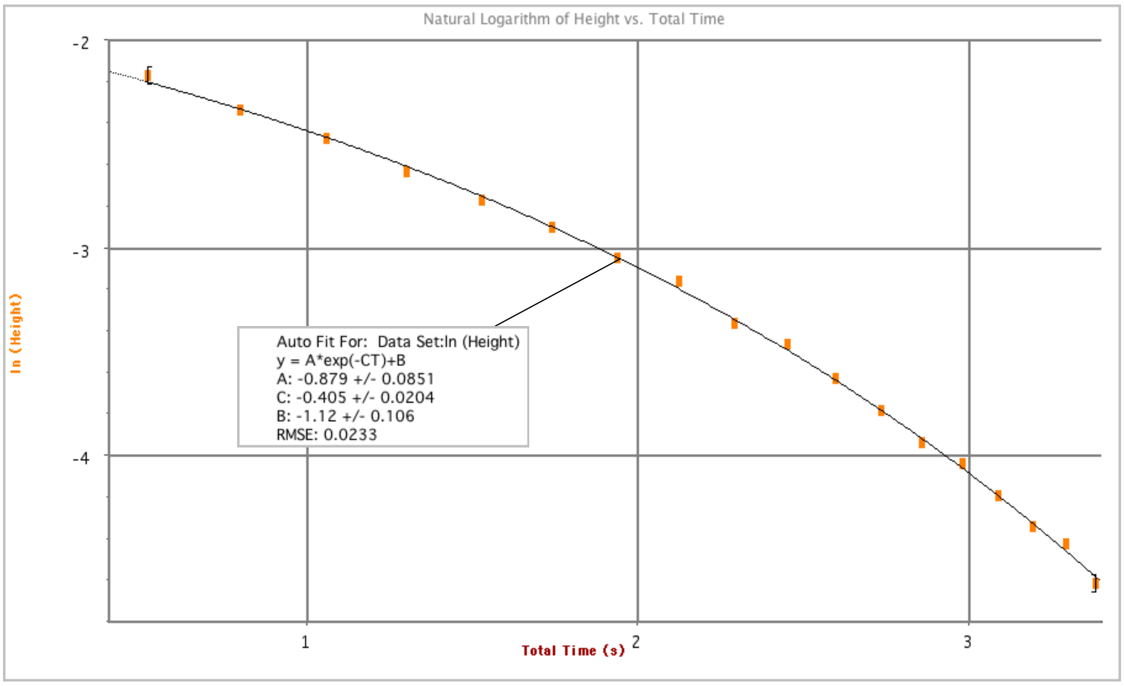
H is graphed against total time (see Figure 4). Error bars have not been added since, as described above, the uncertainty in the calculation of *H* is insignificant.

Figure 4.



Clearly this is not an exponential decay as the time it takes consecutive values of *H* to deduce 0.5 *H*, is not constant. To find a relationship between *H* and *T*, a graph of the natural logarithm of *H* against total time is plotted.

Figure 5.



The graph shows that there is not a power relation between *H* and *T*. It was at this stage that it was appreciated that the wrong research question was being asked. Since time intervals between bounces are not the same, the research question should be "Does the height *H* of consecutive number of bounces made by the ball decay exponentially with bounce number?" Hence a graph of *H* against bounce number is plotted and shown below.

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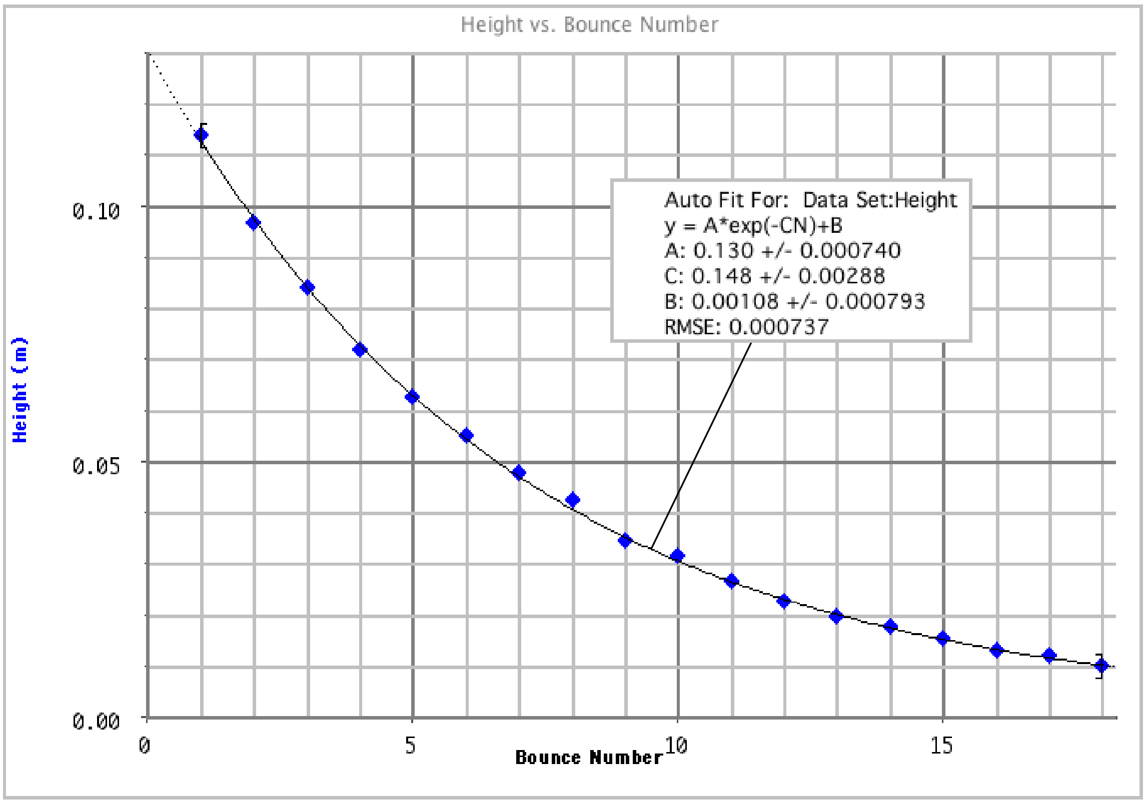
Figure 6.



H reduces from 0.10 m to 0.05 m in about 5 bounces (4.9) and then from 0.05 to 0.025 in a further 5 bounces, thus indicating an "exponential decay".

Assume that $H = H_0 e^{-\lambda n}$ where λ is the decay constant and n is the bounce number so that a plot of the natural logarithm of H against n should give a straight line graph the gradient of which is $= \lambda$. This graph is plotted below.

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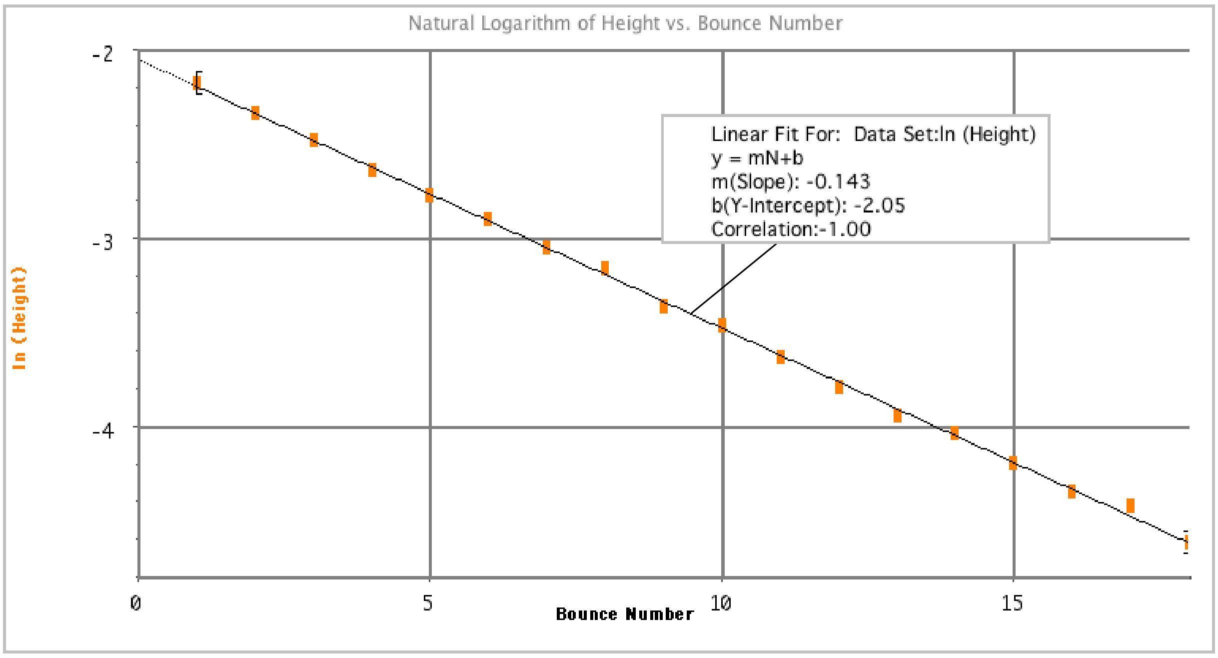
Figure 7.



HALF-LIFE ANALYSIS

The gradient of the graph is calculated by the computer as $m = \lambda = -0.143$. The "half-life" is calculated from $t_{\frac{1}{2}} = \dfrac{\ln 2}{\lambda} = 4.85\,\text{bounces}$. This ties in with the plot of H against bounce number (see above).

CONCLUSION AND EVALUATION

Results: The results show that, for this particular ball and surface, the height of consecutive bounces decays exponentially with the number of bounces. However, one has to remember that the decrease in height is a set function. As such, the decay would only be true if there were a very large number of bounces and after each bounce there was a very small decrease in height.

Limitations: The only limitation in this experiment is that there was insufficient time to take more data. More data would have helped to test the validity and or limitations of the exponential rule in this situation but also to test its validity for balls of different material dropped into different surfaces.

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Improvements: The ball could have been dropped from a greater height to increase the number of data points and dropped from a smaller height to decrease the number of data points. This could have been repeated for different balls dropped on the different surfaces from different heights.

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