

Investigating the Change in Acceleration of a Trolley Running Down an Inclined Plane

The Investigation

Research Question: Is the acceleration of a trolley down an inclined plane constant?

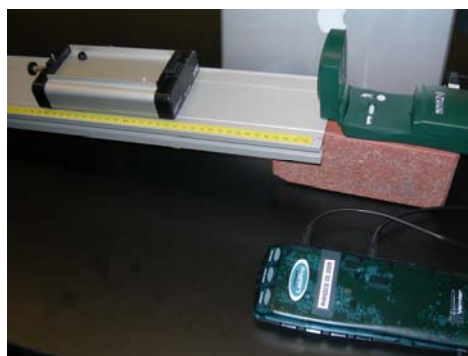
The **time** the trolley travels is the **independent variable** while the **distance** it moves is the **dependent variable**. The **controlled variables** are the equipment settings, the physical set up of the runway and trolley (initial height, release method, etc.), and the room temperature. Times and distances are measured by an ultrasonic motion detector and computer software. The distance is determined from echo time and the speed of sound.

Uniform acceleration is related to distance s and time t by the equation $s = \frac{1}{2}at^2$.

A graph of distance against time squared will be a straight line with gradient equal to $\frac{a}{2}$.

Equipment

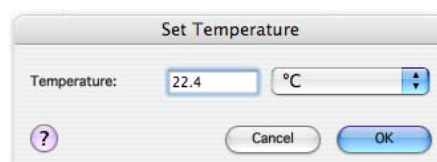
The interface unit was a **LabPro** connected to a **Motion Detector 2** (Model MD-BTD), both by Vernier. See the photo on the right. The software was Vernier's **Logger Pro** version 3.4.1. The runway is a standard lab aluminum one-meter ramp, and the trolley is a PASCO low-friction trolley (with ball bearing wheels). I set the runway up on a brick of about 10 cm height. Tape was used to secure the ramp.



Uncertainties: Issues of Resolution, Precision and Accuracy

Calibration. The **accuracy** (a value compared to a known standard) of the Motion Detector depends upon the room temperature. Because the Motion Detector uses the speed of sound to determine distance, and the speed of sound depends on the air temperature, then the temperature of the air during the experiment must be measured. The motion detector can easily be **calibrated** to the room temperature.

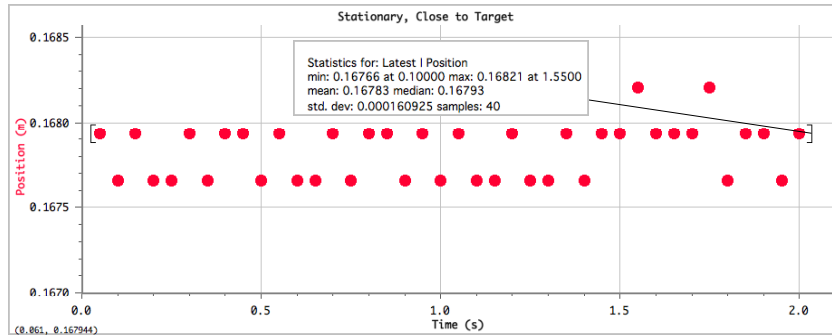
The room temperature at the time of the experiment was 22.4°C. This is used to calibrate the Sonic unit.



Timing Accuracy. In the Vernier unit the timing rate is 1.00 MHz for a period of $\Delta t = 10^{-6}$ s. Uncertainties in timing can be ignored. The speed of sound in air at 22.4°C is 342 ms^{-1} . In 10^{-6} s, sound will therefore travel 3.42×10^{-4} m. In fact, the distance is half this amount because the sound wave is reflected back to the detector therefore

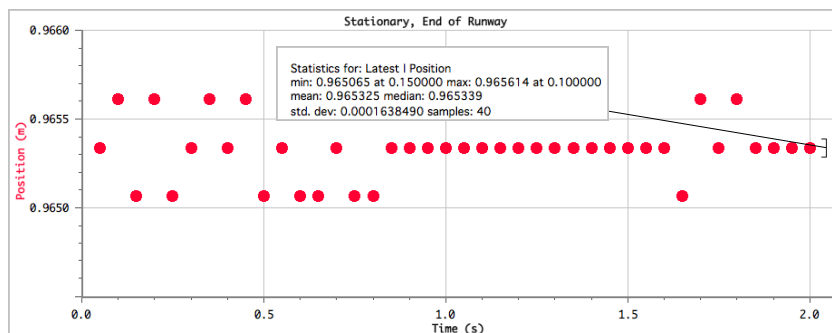
the **resolution** (the minimum detectable change) is about 0.1 mm. Vernier claims a resolution of 1 mm.

Precision of Measurements. I will use the scatter of measurements to determine the uncertainty in the sonic unit. The sample graph below show the range of values for a stationary target close to the sonic unit.



The first and second decimal places of positions are all identical. Only by the third decimal place do we detect some variation. The maximum value is 0.176821 m and the minimum is 0.16766 m, with the median value of 0.16793 m. The range is 0.00055 m and half the range is 0.000272 m or about ± 0.0003 m. This is a precision of ± 0.3 mm. The stationary target is thus measured to be (0.1679 ± 0.0003) m or the uncertainty is ± 0.3 mm.

With the stationary target placed at the end of the runway, the following data is recorded.



Here the maximum is 0.965614 m, the minimum is 0.965065 m, and the median is 0.965339 m. The range is 0.000549 m and half the range is 0.00027 m, or an uncertainty of ± 0.0003 m. Again, it is about ± 0.3 mm.

Therefore, for both the near and far targets, the **precision** (or self-repeatability) of the ultrasonic range system is established as ± 0.3 mm, or ± 0.0003 m.

Speed and Acceleration. Since speed is calculated from changes in consecutive relative positions, speed values do not need calibration; only the distance uncertainty needs to be propagated.

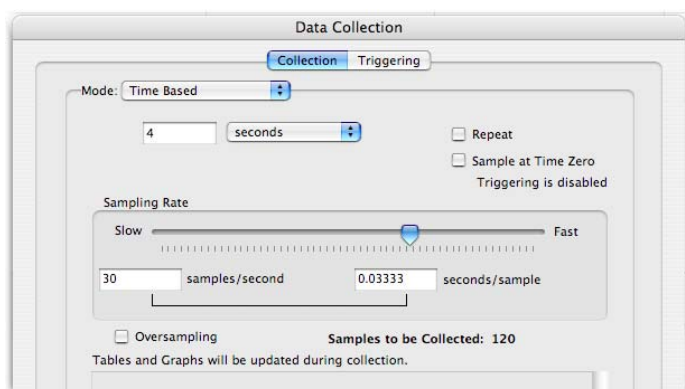
Systematic Uncertainty. There is another source of measurement error. In the time interval for the ultrasonic pulse to reflect off the trolley and travel back to the sensor unit, the trolley will have moved slightly forward. The uncertainty here is not constant but should increase in a linear way as the distance increases. This factor of uncertainty may be ignored since the motion of the

trolley is relatively slow and the overall range is small. Moreover, systematic shifts in speeds against times will not matter when calculating acceleration from the gradient of a graph.

Overall Uncertainty. The uncertainty in the longest distance moved by the trolley is ± 0.3 mm. The systematic shift in measuring technique may be ± 0.1 mm or more, and the calibration for the speed of sound may be ± 0.1 mm or more. Overall, looking at the worst possible case, a general uncertainty of ± 0.7 mm to ± 1 mm in all distance measurements would be acceptable. Hence, Vernier's stated uncertainty of ± 1 mm can be accepted. . Over a distance of 0.5m, the uncertainty is therefore $\pm 0.2\%$. The timing uncertainty in an interval, say of 2s, is only 0.00005%. Hence timing uncertainty may be ignored.

Setting Up the Detection Unit and Software Settings

After trialing different sample rates, it was found that a frequency of 20 Hz (for a period of 0.05 s) worked well. A sample time of 5 seconds was also selected. The Data Collecting box (as shown here) was adjusted to these values.



Data

The data-logging process recorded the raw data of time and position. Here is a sample of the data as collected by the data logging.

Time (s)	Position (m)	Time Squared (s ²)
0.050000	0.1899	0.002500
0.100000	0.1896	0.010000
0.150000	0.1899	0.022500
0.200000	0.1896	0.040000
0.250000	0.1899	0.062500
0.300000	0.1902	0.090000
0.350000	0.1902	0.122500
0.400000	0.1913	0.160000
0.450000	0.1943	0.202500
0.500000	0.1981	0.250000
0.550000	0.2033	0.302500
0.600000	0.2105	0.360000

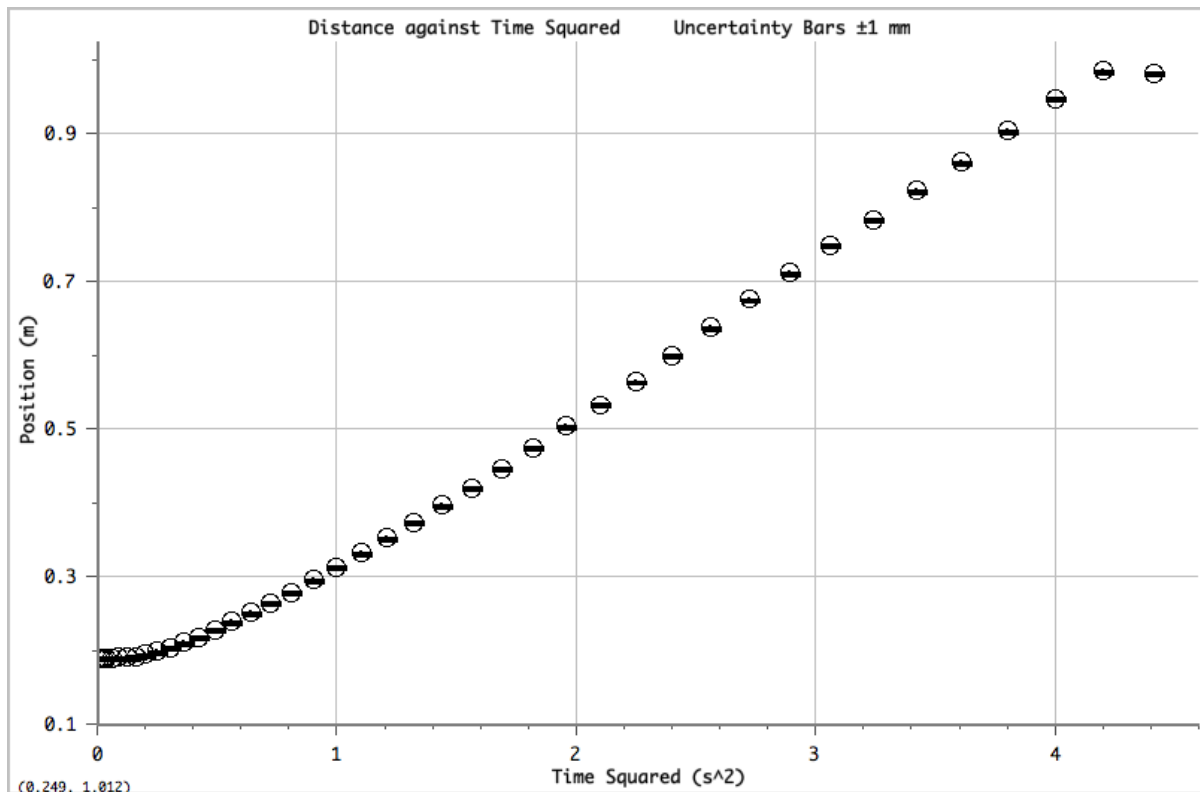
Data table headings would ideally be as show below.

<i>Time</i> <i>t / s</i> $\Delta t \approx \pm 0 \text{ s}$	<i>Distance</i> <i>s / m</i> $\Delta s = \pm 0.0003 \text{ m}$	<i>Time Squared</i> <i>t² / s²</i> $\Delta t^2 \approx \pm 0 \text{ s}^2$
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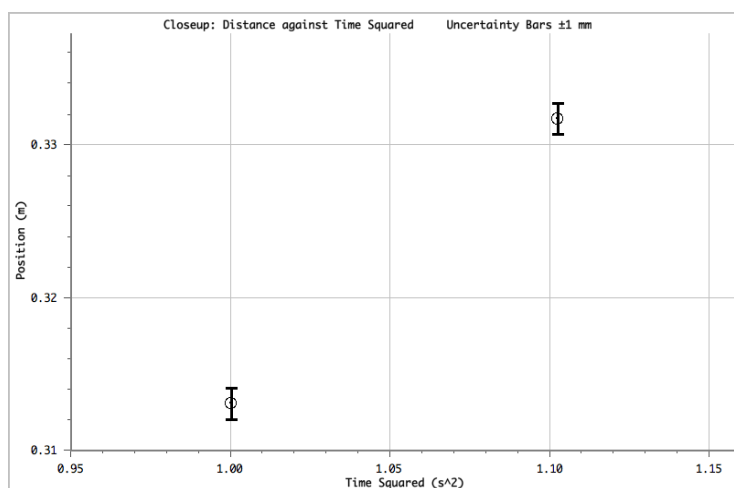
Squaring is a simple data processing function, where, for example, using the 4th data information, $t^2 = t \times t = 0.20 \text{ s} \times 0.20 \text{ s} = 0.04 \text{ s}^2$.

Data Analysis

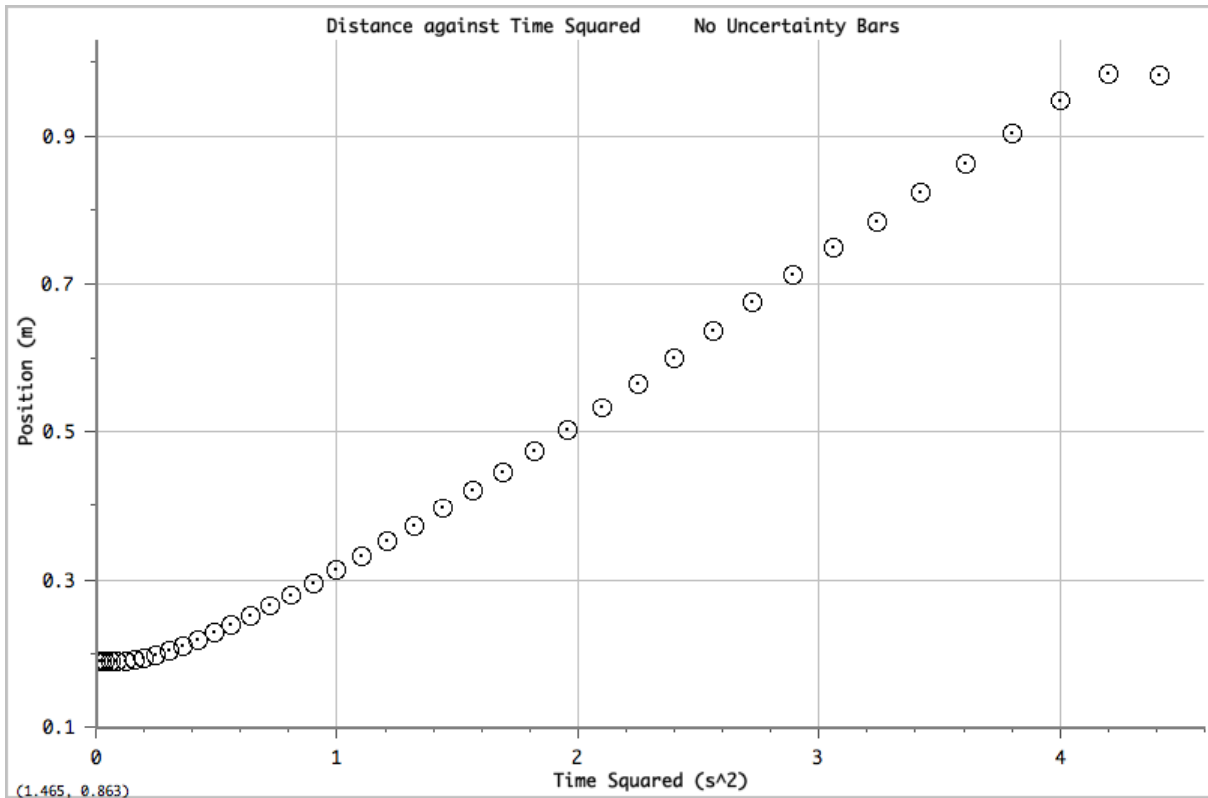
Below is a graph of position against time squared with uncertainty bars. The error here is negligible. The uncertainty bars look funny because they are so small, the hat and the trail overlap given the distance scale.



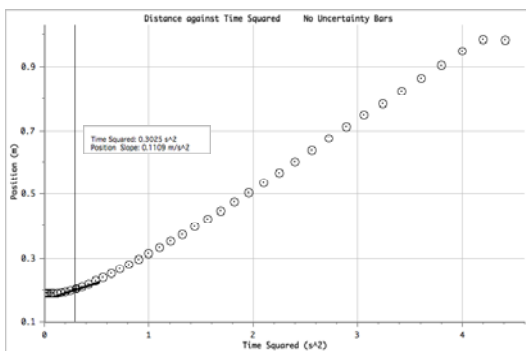
Here is an enlargement of a section of the graph with uncertainty bars at ± 1 mm. The uncertainty bars are insignificant. This is little point in trying to construct maximum and minimum gradients.



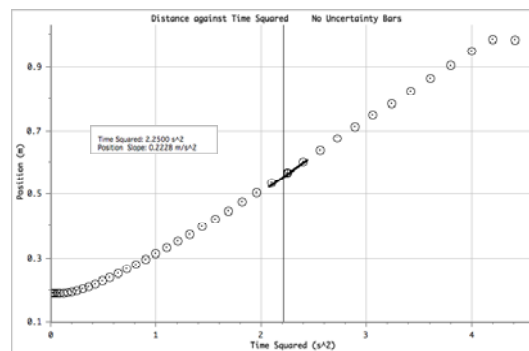
Here is the main graph.



The Tangent Tool was used to find the gradient at various data points. An example of a range where the acceleration is not uniform and where it is uniform, are shown below.

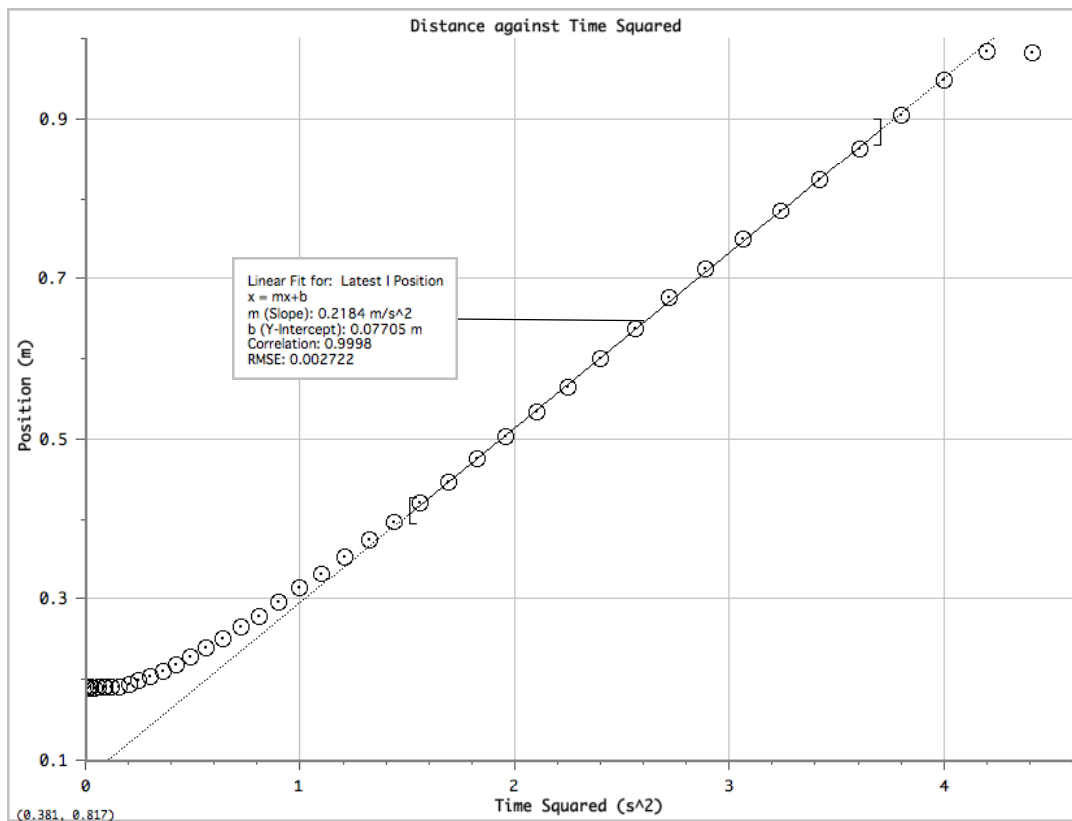


Changing acceleration region.



Uniform acceleration region.

The graph below is used to calculate the gradient of a linear region of the graph. The straight line is interpolated to highlight the region of non-uniform acceleration.



Using the above graph, the uniform acceleration a (for the selected data) is given by

$$a = 2 \times \text{gradient} = 2 \times 0.2184 \text{ m s}^{-2} = 0.4368 \text{ m s}^{-2}.$$

As described above, the uncertainty in the gradient is 0.2%, so the uncertainty in the acceleration is 0.4%.

$$a = 0.4368 \text{ m s}^{-2} \pm 0.4\% \rightarrow a = (0.4368 \pm 0.0017) \text{ m s}^{-2} \approx (0.437 \pm 0.002) \text{ m s}^{-2}$$

The experiment was repeated several times under identical conditions. The following table summarizes the results.

Trial	2 x Gradient of Distance against Time Squared / ms^{-2}
1 (shown above)	$2 \times 0.2184 = 0.4368$
2 (not shown)	$2 \times 0.1932 = 0.3862$
3 (not shown)	$2 \times 0.2148 = 0.4296$
4 (not shown)	$2 \times 0.1677 = 0.3354$
5 (not shown)	$2 \times 0.2120 = 0.4240$
Average	0.4024
$\text{Range} = \frac{(0.4368 - 0.3354)\text{ms}^{-2}}{2} = \pm 0.0507\text{ms}^{-2} \approx \pm 0.05\text{ms}^{-2}$ $a_{\text{average}} = (0.40 \pm 0.05)\text{ms}^{-2} = 0.40\text{ms}^{-2} \pm 13\%$	

Conclusion and Evaluation

Uniform Acceleration. The calculated uniform acceleration based on one trial would seem to be very precise, viz, $a \approx (0.437 \pm 0.002)\text{ms}^{-2}$.

Repetition however, reveals a much less precise result. The values are scattered and the result calculated from a range of these five trials is only accurate to two significant figures.

$$a_{\text{average}} = (0.40 \pm 0.05)\text{ms}^{-2}$$

The quality of the measurements is reduced by the scatter of gradient values in the multiple trials. Therefore the uncertainty of 13% should be accepted.

Changing Acceleration. Analysis of the distance against time-squared graph shows uniform acceleration in the range from about 1.56s^2 to 3.61s^2 . After 3.61s^2 (or about 1.9s) the trolley collides with the end of the runway.

The acceleration is non-uniform in the range from start to about 1s. This could be because the frictional force acting on the trolley is varying in this range and then becomes constant.

Weakness and Improvements. There are two weaknesses in this investigation. First, the variation in trials suggests that there are factors that need to be better controlled. Perhaps the release mechanism could be improved. An electromagnet could be used to hold the trolley in place and then gently release it.

Second, instead of a 10 cm height, a height of 20 or 30 cm for the given 1.2 meter long runway could be used. Alternatively, the same height could be used but with a much longer runway, perhaps 2.5 meters long. Having a greater acceleration and/or increasing the range over which measurements are taken would help reduce the effect that small variation in the movement of the trolley might have on the results.

Finally, it would be interesting to investigate the region of non-uniform acceleration in more depth.