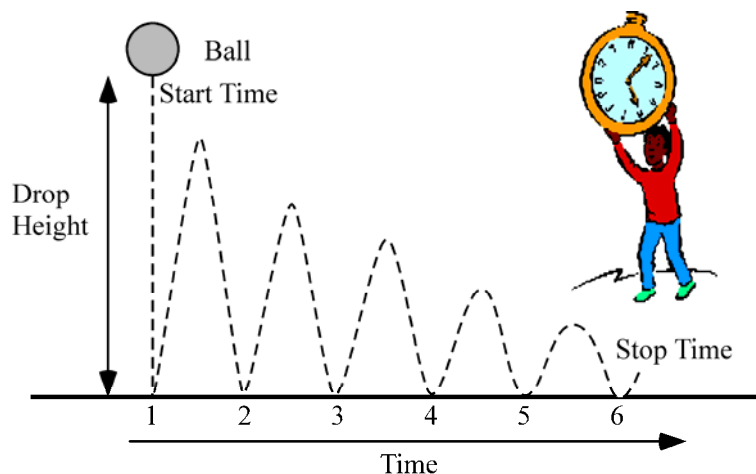**Investigating the relationship between the drop height and time
of 6 bounces of a super-ball.**

INTRODUCTION

In this laboratory I'm going to relate the time that a ball needs for 6 bounces from different dropping heights.

My variables are height, time for bounces, mass of ball, bouncing surface and number of bounces. The independent variable is the dropping height H because I choose it. The dependent variable is the bouncing time T because this depends on the drop height. The constants must be the mass of ball, bouncing surface, and number of bounces because they are going to be the same during the whole experiment.

The question to answer is how the time of six bounces is related to the height of dropping it. I will look for a linear and proportional relationship between the independent and dependent variables. My idea is if the height increases, the time will increase. If I don't find this result I will graph whatever is needed to find the relationship.

Time Six Bounces against Dropping Height
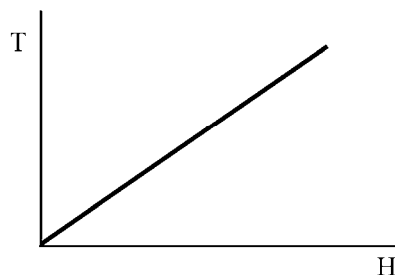
Therefore the function of this graph would be $T = mH$ where T is time, m is the gradient, and H is the height.

DESIGN

The method to make this experiment is easy and simple. The equipment and materials that I will use are: one ball, a stopwatch, a meter stick, the floor surface, a table, and materials to write.

When I have all of this, I will start the measure of the independent variable. I'll use a paw of a table and there I'll mark lightly different heights with the ruler. I'll start with 20 and then 30, 40, 50, 60 and I'll use the height of the table and the ruler too.

To make the experiment I will put the ruler horizontally to the mark in the table and in the edge of the rule I will put the ball.

Then I'll leave the ball fall, therefore now I'm going to explain how I will measure the time (dependent variable). When the ball is on the ruler I'll be ready with the watch in my hand. I'll leave the ball go from the ruler and I will press the button on the stopwatch at this moment in order to start timing. I will then watch and listen for the ball to make 6 bounces. At the moment of the 6th bounce I will stop the stopwatch timing.

Also I should explain how I'm going to keep the constant variable. The surface that I'll choose it'll be the ground of the classroom and the ball will be a showy ball and therefore I will not miss the ball.
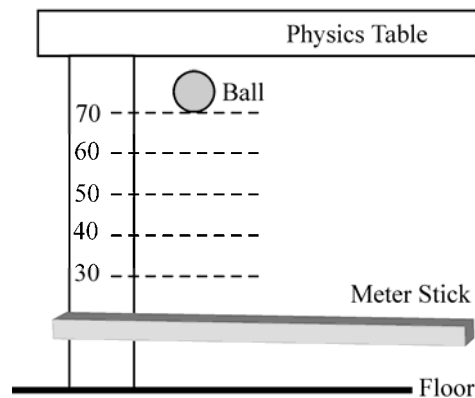
Another point to talk about is how many times I'll measure the variables. I will measure the time for six bounces 4 times for each different height and then I'll take the average. To measure the height I'll repeat it 3 times and I'll take the average. I'm going to take 7 different values from 20 cm being the smallest height until 100 cm being the longest height.

DATA AND ANALYSIS

With all of the process done I have to measure and write the values, therefore I make the raw data table here.

<div align="center">Raw Data</div>

#	Drop Height H (cm) Uncertainty ± 0.2 cm			Time of 6 Bounces T (s) Uncertainty ± 0.2 s			
1	20.0	19.0	20.0	1.68	1.78	1.79	1.57
2	30.0	30.0	29.5	1.97	2.10	2.34	2.28
3	41.0	40.0	39.0	2.35	2.46	2.75	2.77
4	50.0	50.5	50.5	2.72	2.73	2.72	2.72
5	60.0	60.1	60.6	3.19	3.01	3.09	3.16
6	77.5	77.9	77.2	3.32	3.28	3.59	3.35
7	100	100.3	100.9	4.03	4.00	3.97	4.03

I estimate the uncertainty in the height as about 0.2 cm. The uncertainty in the bouncing time is harder to figure. Taking the difference between the largest and smallest time for each height I find the range of uncertainty. This is 0.22 s, 0.37 s, 0.42 s, 0.01 s, 0.18 s, 0.31 s and 0.06 s. The average of these ranges is 0.22 s, so half the range is 0.11 s or ±0.1 s. But that is too precise given that five of the ranges are much more than this, so to be safe I say the timing uncertainty is ±0.2 s. This seems reasonable. By the way, I first drew a graph with uncertainty bars at ±0.1 s and the best straight line did not cross many of the uncertainty ranges, so 0.2 seconds is better.

Now I need to process my data to find averages.

For the height, $H_{ave} = \dfrac{H_1 + H_2 + H_3}{3}$. This was done on calculator. I decided to keep the uncertainty here as ±0.2 cm or ±0.002 m. I also changed heights from cm to m.
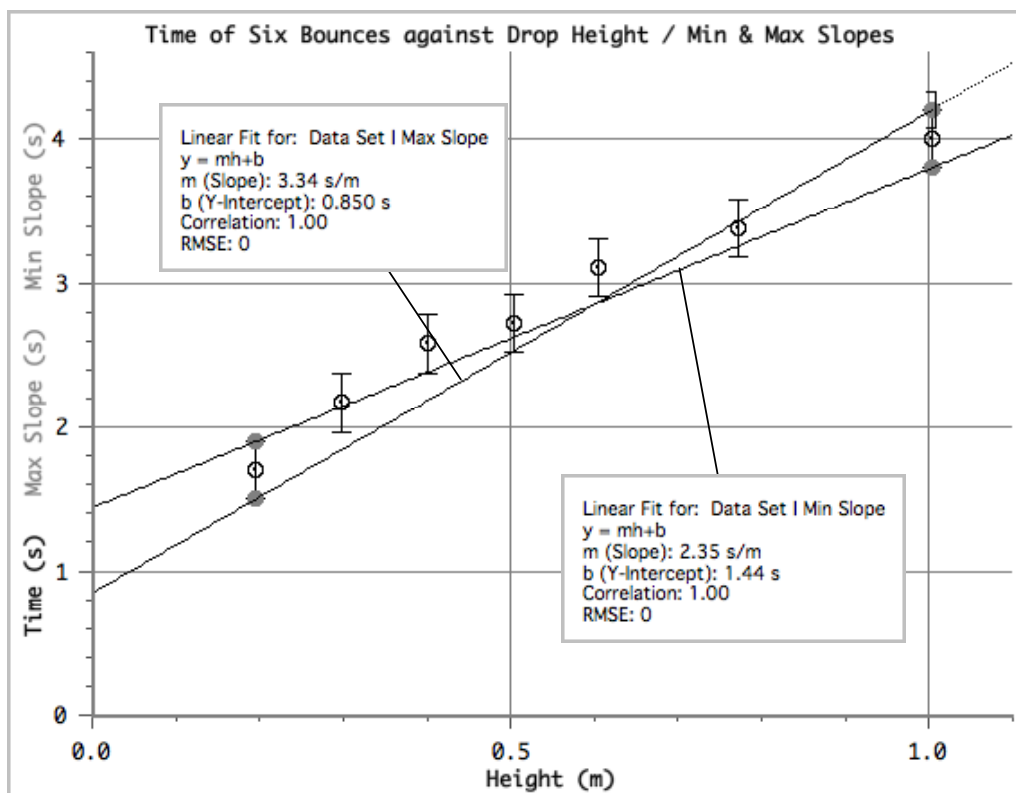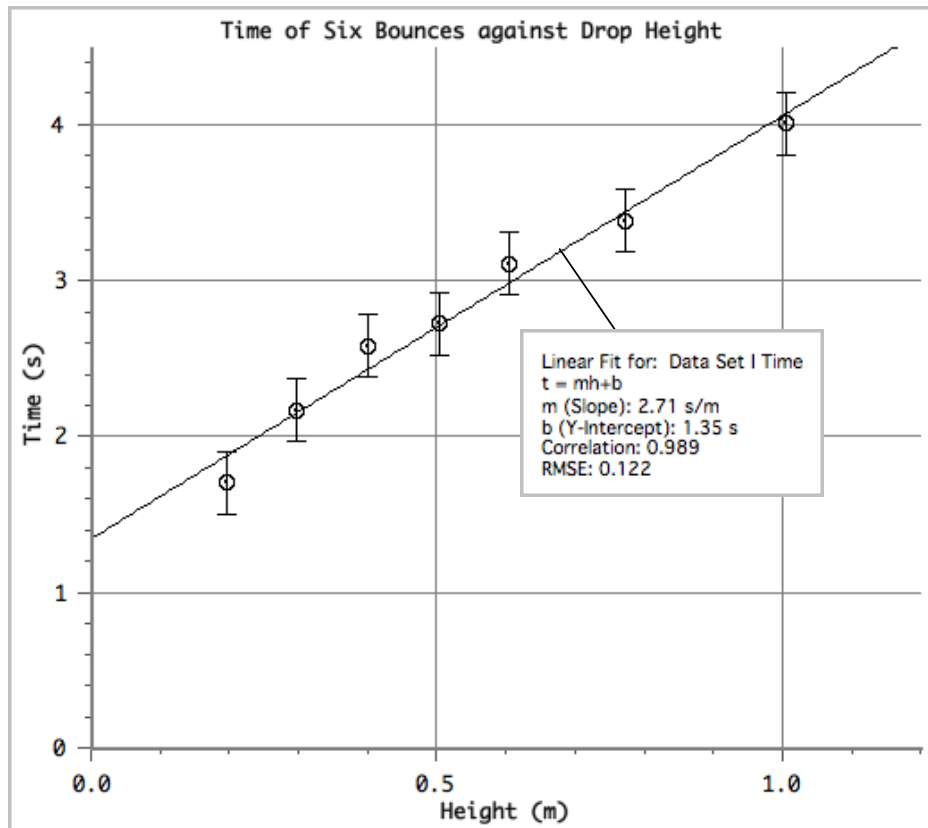
For times, $T_{ave} = \dfrac{T_1 + T_2 + T_3 + T_4}{4}$ and this was done on calculator. Averaging should reduce the uncertainty but because of the variation in uncertainty range for the different heights, I decided to keep the time uncertainty as ± 0.2 s.

I will keep my numbers to three decimal places even though the uncertainty is only one decimal place because I will round off numbers only at the conclusion.

Processed Data / Averages

#	Average Height H (m) $\Delta H = \pm 0.002$ m	Average Time of 6 Bounces, T (s) $\Delta T = \pm 0.2$ s
1	0.196	1.705 ≈1.7
2	0.298	2.170 ≈ 2.2
3	0.400	2.580 ≈ 2.6
4	0.503	2.722 ≈ 2.7
5	0.605	3.110 ≈ 3.1
6	0.772	3.380 ≈ 3.2
7	1.004	4.007 ≈ 4.0

Now I construct a graph of time against height. The uncertainty in the height is relatively small and so I will ignore this, while the uncertainty in the time is more significant and so I will show error bars for time.

Time of Six Bounces against Drop Height

Linear Fit for: Data Set I Time
t = mh+b
m (Slope): 2.71 s/m
b (Y-Intercept): 1.35 s
Correlation: 0.989
RMSE: 0.122

Time of Six Bounces against Drop Height / Min & Max Slopes

Linear Fit for: Data Set I Max Slope
y = mh+b
m (Slope): 3.34 s/m
b (Y-Intercept): 0.850 s
Correlation: 1.00
RMSE: 0

Linear Fit for: Data Set I Min Slope
y = mh+b
m (Slope): 2.35 s/m
b (Y-Intercept): 1.44 s
Correlation: 1.00
RMSE: 0

The best straight-line gradient $m_{Best} = 2.71$, the maximum gradient $m_{Max} = 3.43$ and the minimum gradient $m_{Min} = 2.35$. The range is:

$$m_{Max} - m_{Min} = 3.43 - 2.35 = 1.08$$

Half of this range is the uncertainty in the best straight line.

$$\Delta m_{Best} = \pm\frac{1.08}{2} = \pm 0.54 \approx \pm 0.5$$

The gradient and its uncertainty are thus $m_{Best} = 2.71 \pm 0.495 \approx 2.7 \pm 0.5$

The uncertainty divided by the gradient times 100 gives us an error of about 19%, and that is not good. The correlation between T and H is suspect.

The general equation $y = mx + c$ for my data is now $T = mH + c$ where the proportionality constant $m_{Best} \approx 2.7 \pm 0.5$ and the systematic shift in the line is c, where $c = 1.19\,\text{s}$. My research question said that $c = 0$ but this is not true. Let us re-examine the data in more detail.

PROBLEMS WITH ANALYSIS

I notice at the y-intercept, for a height of zero, a time of 1.35 s. This is impossible, so the systematic shift must have some meaning. Perhaps the time from the release to the first bounce offsets all the data points. So, using the theoretical time from $H = \frac{1}{2}gt_{drop}^2$ to $t_{drop} = \sqrt{\frac{2H}{g}}$ I had

the graphing program calculate the revised bounce time as $t_{revised} = T - \sqrt{\frac{2H}{g}}$ where g is gravity

at $9.81\,\text{m}\,\text{s}^{-2}$. Here is the graph.
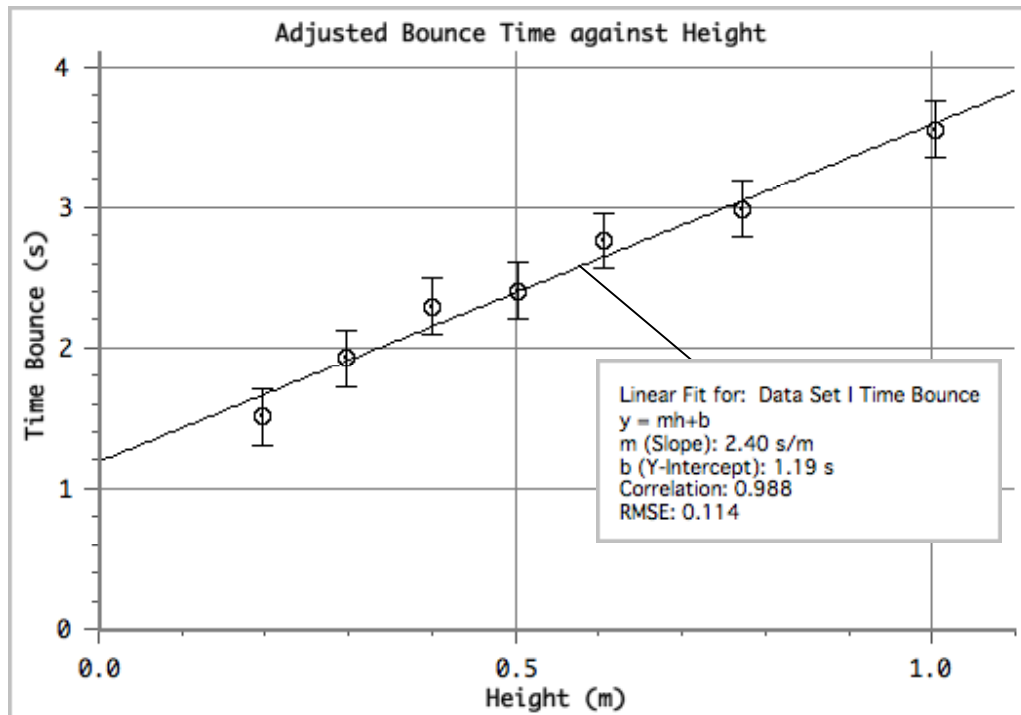
The *y*-axis offset is still significant, about 1.19 s compared to the previous graph offset of 1.35 s. There must be some other theoretical problem.
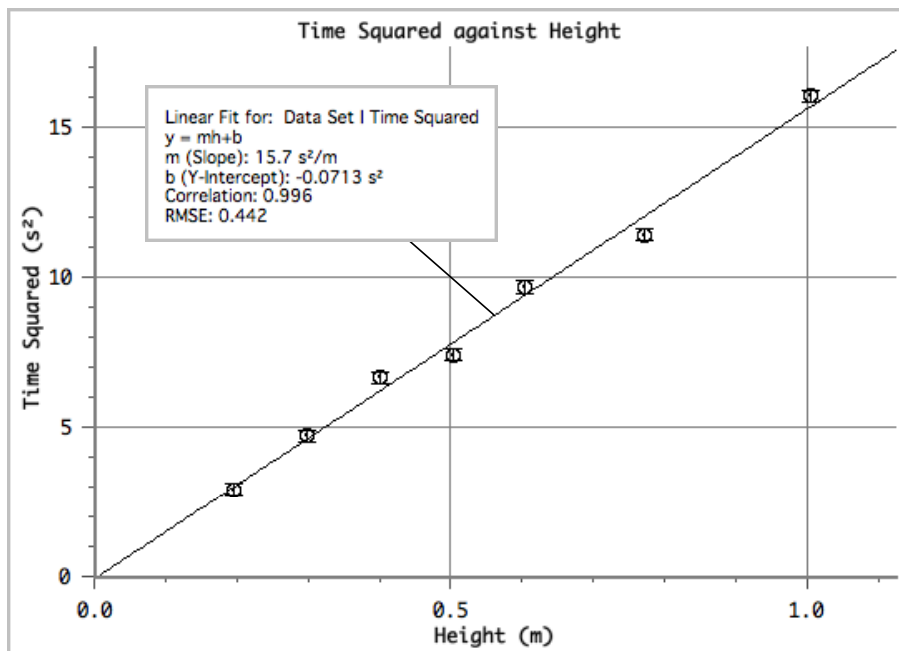
Looking close at the data points and realizing that the time must be zero for a zero height, I might suggest a curving trend in the data. Perhaps the true shape of the graph is not a straight line. Next I try logarithms to find the relationship between time and height. The graphing software does this for me when I define the terms.
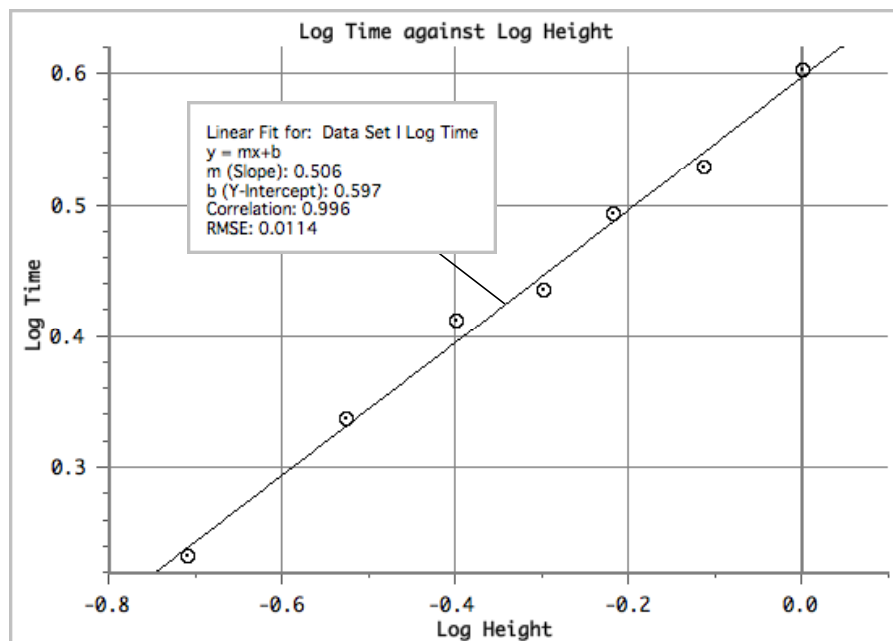
This is great news. There is a high correlation of 0.996 and the gradient is 0.506 or about 0.5. The gradient is the exponent n and the proportionality constant is now k.

$$T = kH^n \quad \rightarrow \quad \log T = \log k + n \log H$$

With logarithms, we can say that $n = 0.5$ or $T \propto H^{0.5}$ which is $T \propto \sqrt{H}$ or $T^2 \propto H$. Here is a graph of time squared against height.

Time Squared against Height

Linear Fit for: Data Set I Time Squared
y = mh+b
m (Slope): 15.7 s²/m
b (Y-Intercept): -0.0713 s²
Correlation: 0.996
RMSE: 0.442

This is great. The best straight lines nearly goes through the origin with an offset of only -0.07 s^2. We can ignore this experimental error. Also, the correlation is 0.996, slightly better than other graphs. I think it is safe to say that my data shows that $T^2 = mH$ and not $T = mH$.

CONCLUSION

In my conclusion I am going to relate what I got and what I was expected to get. My experiment investigated the relationship between the time that a ball does 6 bounces as a function of dropping height. As the height increases I expected the time to increase. My graph showed this. The graph was linear but did not pass through the origin. I suspected some systematic error in the theory. Although I was more or less correct, I also sense a trend in the graph as the data kind of curved. I then graphed the log of time against the log of height and found that the graph of time squared against height was a much better straight line and it went through the origin. Hence my original idea is wrong and a new discovery was made, namely that time squared is proportional the drop height.

The most important problem here is in the trend of the data as seen as the scatter of data on the graph. To improve the quality of data and hence to better find the correct trend I would consider the following things.

I would construct a better ball release mechanism, and not do it by hand. Perhaps a clamp and a stand, where the clamp when opened would release the ball without any twisting or turning and the stand would allow repeated drops from exactly the same height.

There is a difficulty in measuring the time of 6 bounces. I could use a computer and data logging equipment to record the sound as time goes by. The bounces would review spikes in the sound level, and the time scale would be very accurate. This would be a great improvement.

I would like to have a wider range of data, perhaps up to 1.60 meters. I would also want more data points within this range, say every 10 cm.

Perhaps the bouncing ball could be restrained in a closed box to keep it from moving off to one side. However, this would also take energy away from the ball and invalidate my data.

There is no known textbook answer on the relationship of time to time of a number of bounces and the drop height, but my discovery of $T^2 \propto h$ must be hidden in the theory of free fall motion and the equations we learned in class. We know (with uniform acceleration) that the impact speed is proportional to the square root of the drop height, and that the bouncing time should be proportional to the impact speed.